Prescriptive Analytics in Health Care

International Summer School: Economic Modelling in Health Care

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City of Wuppertal

http://www.mygeo.info/landkarten/deutschland/Deutschland_in_Europa.png

City of Wuppertal

- Approx. 350,000 inhabitants
- Area of 168.41 km²
- Density: 2,100/km²
- Most famous for its suspension railway ("Schwebebahn")
- University funded in 1972

Source: https://upload.wikimedia.org/wikipedia/commons/thumb/3/30/Schwebebahn_G15.jpg/1200px-Schwebebahn_G15.jpg

University of Wuppertal

- University funded in 1972
- 3 different locations in the city
- 7 different faculties
- 1,800 employees
- Around 22,000 students:
  3,200 students in Business & Economics
- B.Sc. in Health Economics and Management (M.Sc. programme in preparation)

BKG

BERGISCHE UNIVERSITÄT WUPPERTAL

Background

- **Education**
  - M.Sc. in Management & Economics
  - PhD on mathematical optimisation for operating room planning

- **Professional Appointments**

2013 - 2014	Consultant to Medical Director, Städtisches Klinikum Solingen, Germany
2014 - 2017	Postgraduate Research Associate, University of Exeter, UK
Since 2017	Assistant Professor of Operations Management, University of Wuppertal, Germany

- **Research Interests**
  - Develop and apply optimisation & simulation models to solve healthcare problems
  - Emergency Departments, inpatient wards, operating rooms, ambulance locations, …

Outline

- **Lecture**
  - Analytics – What is it and how can we use it?
  - Models & Methods – Distinction and possible uses
  - Application: Decision problems in location and operating room planning
- **Tutorial – Part I**
  - Software R and R Studio
  - Location analysis
- **Tutorial – Part II**
  - Operating room planning
  - Discussion/Presentations

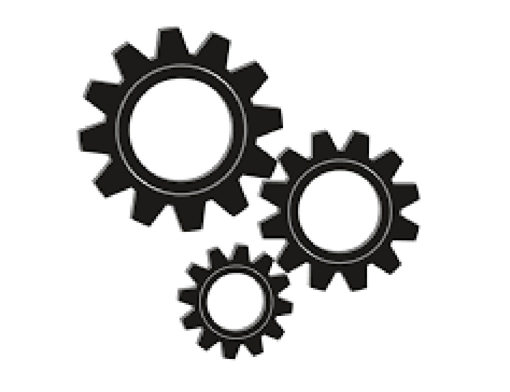
Intended Learning Outcomes

- Distinguish different dimensions of Analytics
- Understand how prescriptive analytics can be applied to health care
- Understand existing mathematical optimisation models
- Independently develop simple optimisation models
- Solve optimisation models using software package R and solver
- Develop, solve and present case study results

Assessing prior knowledge

- Analytics: Can you give a definition?
- Optimisation: what is it?
- R and R Studio: Have you ever used it?
- Optimisation in MS Excel: Do you have any experience with it?

Analytics – What is it?

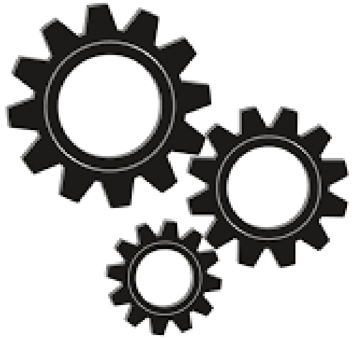
Descriptive Analytics	Predictive Analytics	Prescriptive Analytics
		
What did happen?	What will happen?	What should we do?

Analytics – What is it?

Descriptive Analytics

Predictive Analytics

Prescriptive Analytics



Provide data/structure

Provide data/structure

Predict performance
of decision, revise etc.

Analytics

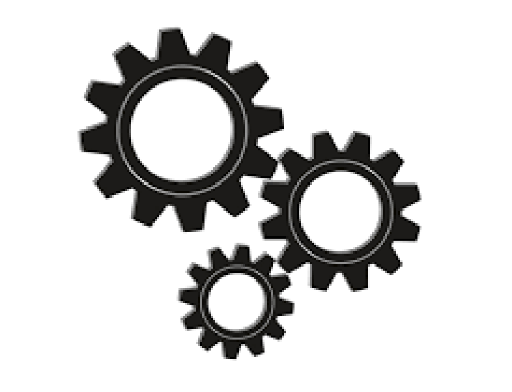
- **Descriptive Analytics**
  - Understanding current issues is key to any further analysis
  - Solving real-world problems requires understanding real-world data

- **Predictive Analytics**
  - How are current issues to develop?
  - What will happen following changes?

- **Prescriptive Analytics**
  - What should you do, given your objective
  - Best possible solution (i.e. optimum) versus real-world setting

Analytics

Descriptive Analytics



What did happen?

Predictive Analytics

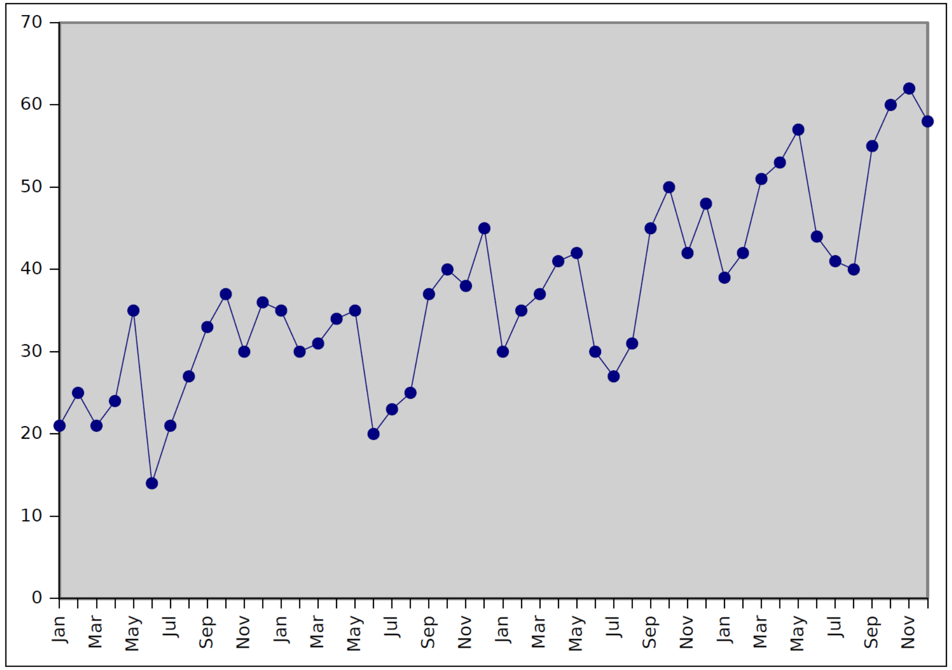
What will happen?

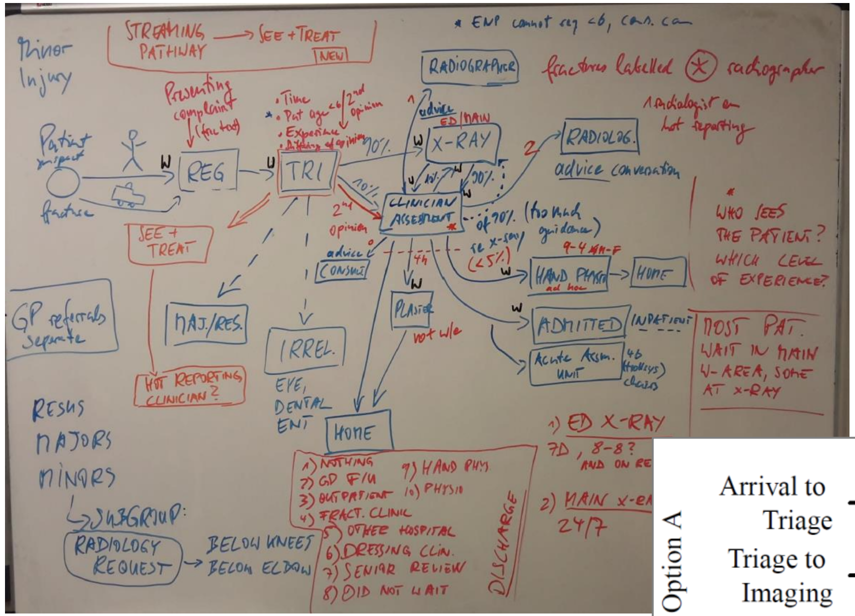
Prescriptive Analytics

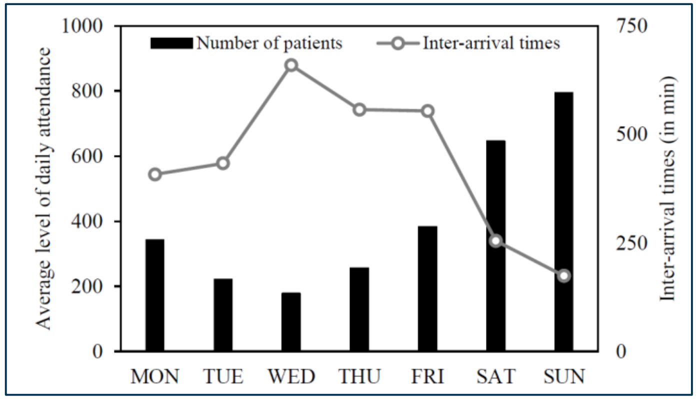
What should we do?

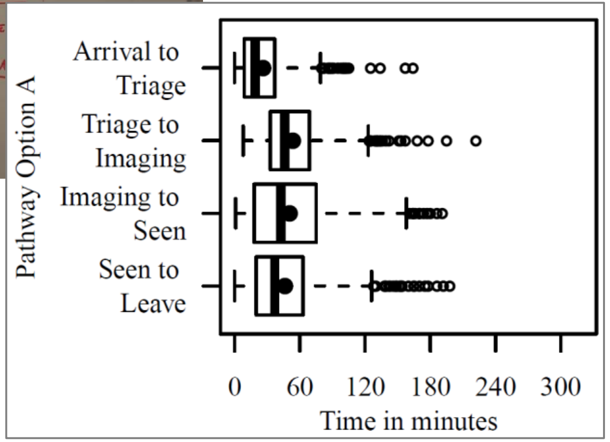
Descriptive Analytics

- Typical examples could be:

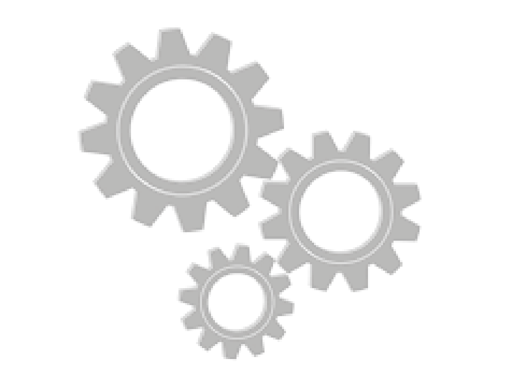






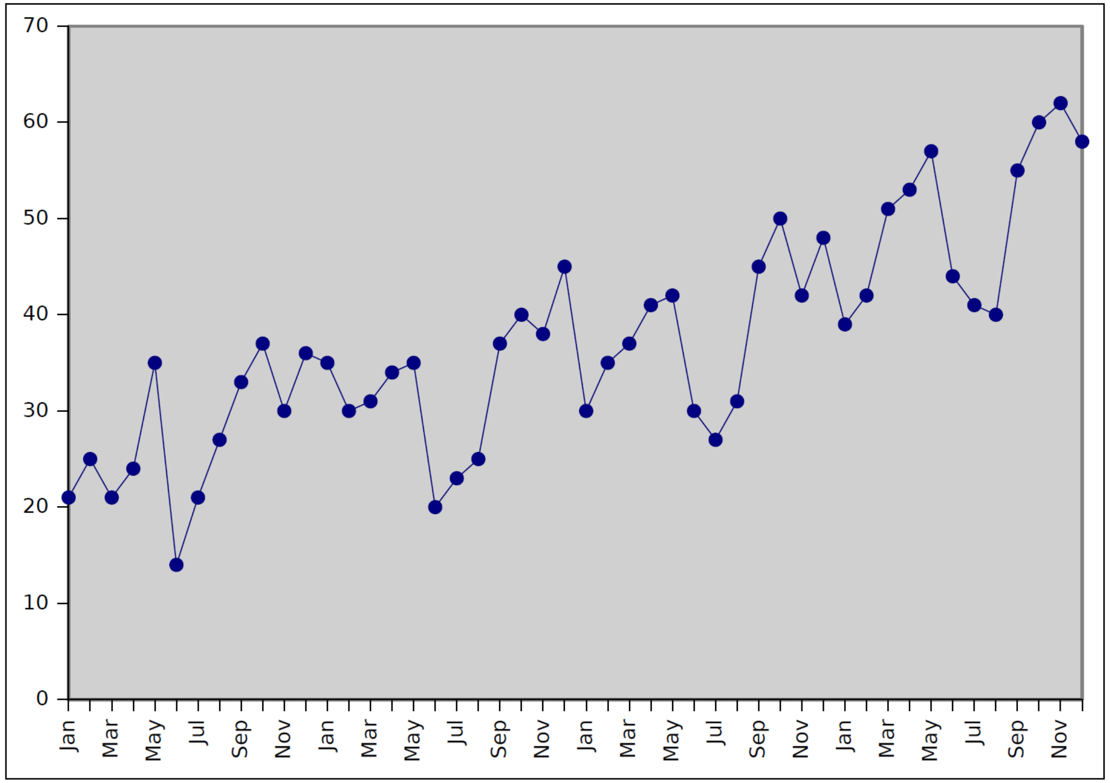


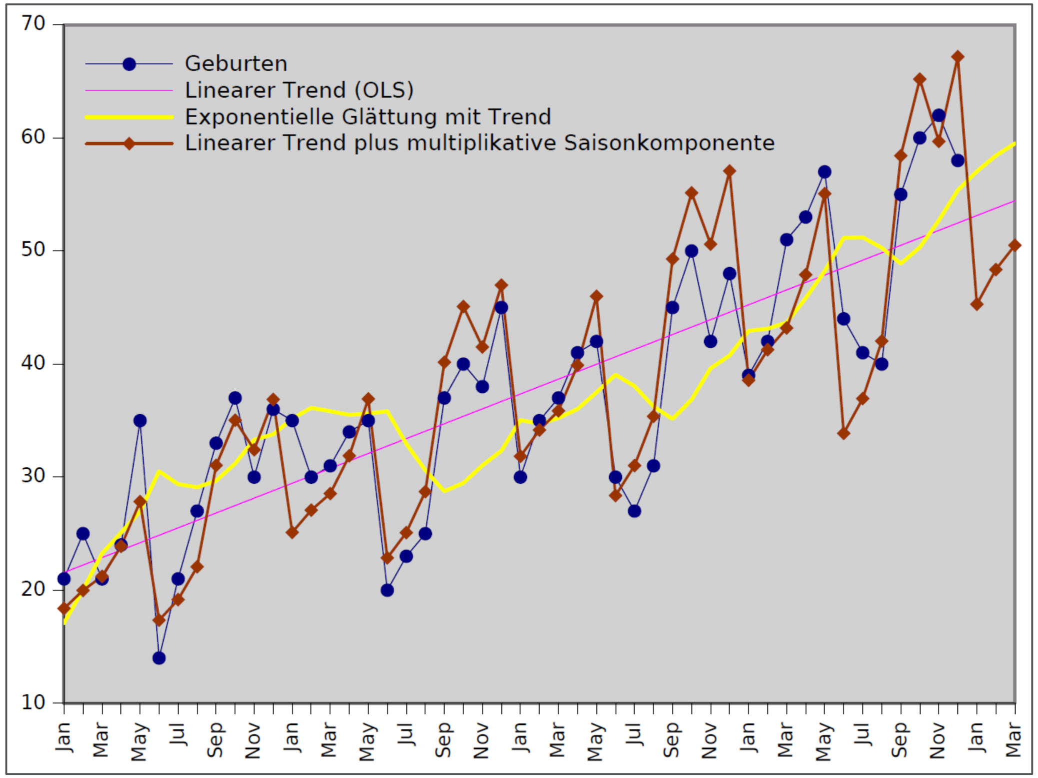
Analytics

Descriptive Analytics	Predictive Analytics	Prescriptive Analytics
		
What did happen?	What will happen?	What should we do?

Predictive Analytics

- (Likely) common example: Time series and prediction models





… other possible tool: Simulation

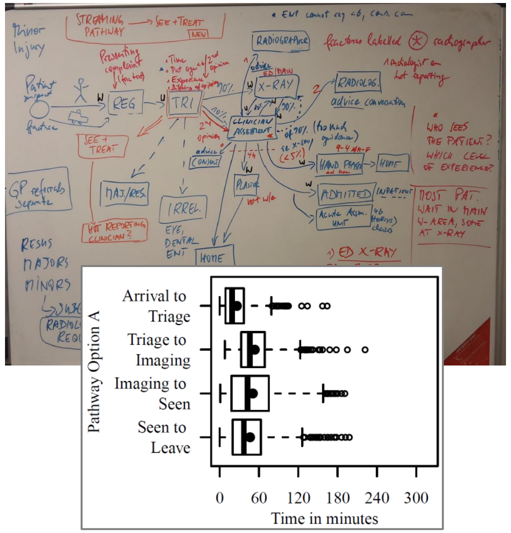
Predictive Analytics

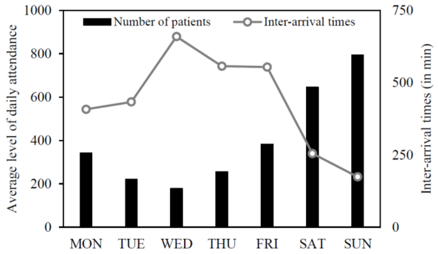
Simulation

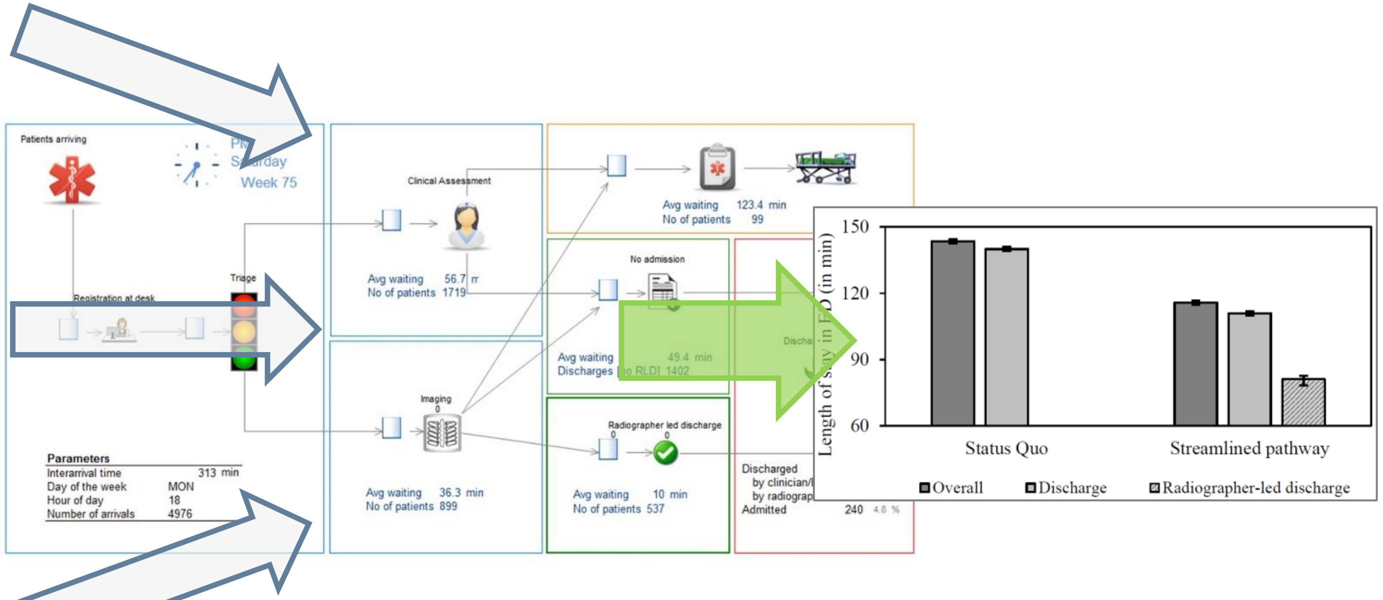
- Basic idea: replicate behaviour of “systems”
- Goals: (1) to explain how things work, (2) to predict likely effects of changes to the system
- Different paradigms:
  - System Dynamics
  - Agent-based Simulation
  - Discrete Event Simulation
    - Tool to replicate pathways and patient flows, e.g. in EDs
    - Dedicated software packages, such as Simul8, ARENA, etc.
    - Manually coded “from scratch”

Predictive Analytics
Simulation

- Example: Assess the likely impact of the introduction of radiographer-led discharge at a hospital in the South West of the UK
- Means: Computer simulation model built to replicate processes in ED



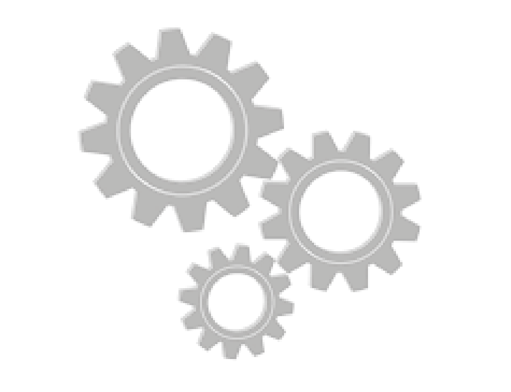




Rachuba, S.; Ashton, L.; Knapp, K.; Pitt, M. (2018): Streamlining pathways for minor injuries in Emergency Departments through radiographer-led discharge, in: OR for Health Care (in press)

Analytics

Schumpeter School of Business and Economics

Descriptive Analytics	Predictive Analytics	Prescriptive Analytics
		
What did happen?	What will happen?	What should we do?

Prescriptive Analytics

What is it?

Answers questions such as:

“What is the best possible decision?”

“What should be done to achieve a goal?”

Prescriptive Analytics

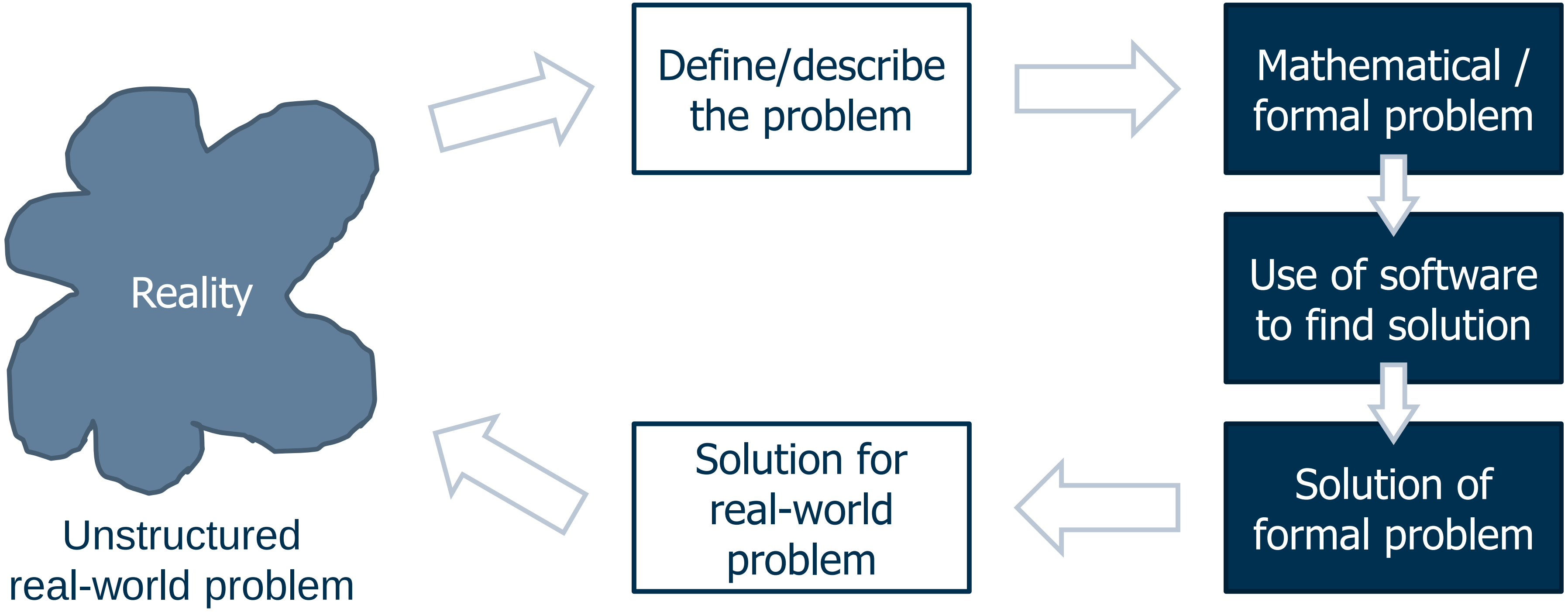
In what situation do we want to prescribe?

- Decisions are often complex because of multiple dependencies (hard to process in your head)
- Dependencies can lead to counter-intuitive decisions – one would normally not expect
- To see whether a decision is "good" is not always trivial (many "almost good" decisions)

Can you think of examples (in health care)?

Prescriptive Analytics

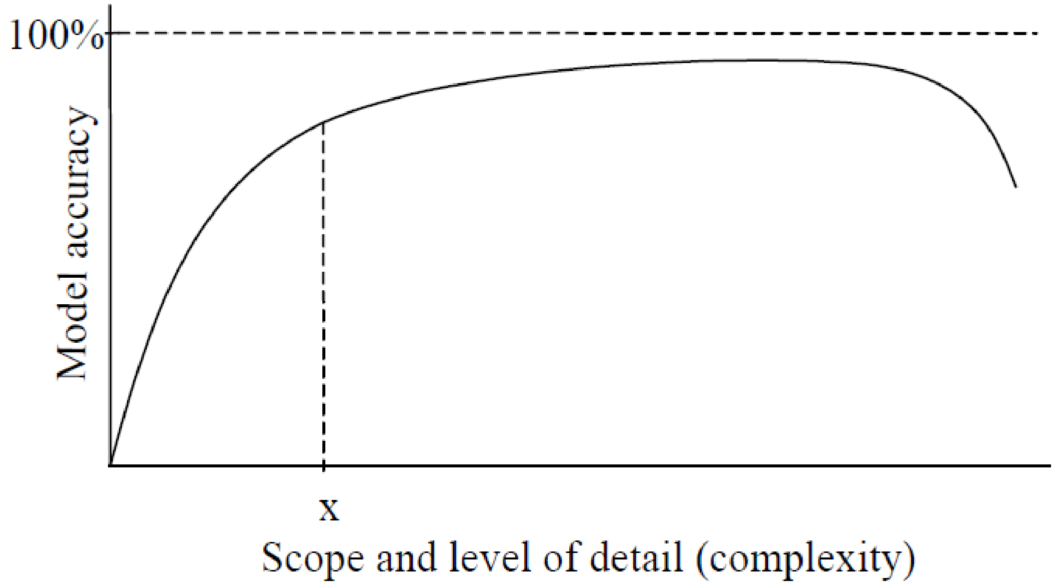
Model-based decision making:



Prescriptive Analytics

A few characteristics of models:

- Models to describe, explain, or decide
- Given purpose / application
- Simplified version of reality
- As much detail as necessary
- As little detail as possible
- Structure is replicated



Source: Robinson, S. (2008).

<u>Example</u>: mathematical optimisation models

Optimisation models

Prescriptive Analytics

Optimisation in a nutshell

- Three major parts:

1. **Decision variables**
   What are possible decisions?

2. **Constraints**
   Are there any restrictions?

3. **Objective function**
   What is the overall goal?

$$10x_1 + 10x_2 \rightarrow \max! \quad 3$$

$$2x_1 + x_2 \leq 10$$
$$x_1 + 2x_2 \leq 8 \quad 2$$

$$x_1, x_2 \geq 0 \quad 1$$

Prescriptive Analytics

Optimisation in a nutshell

- Three major parts:

  1. **Decision variables**
     What are possible decisions?

  2. **Constraints**
     Are there any restrictions?

  3. **Objective function**
     What is the overall goal?

3

$$\max \sum_{s\in\mathcal{S}}\sum_{j\in\mathcal{J}}\sum_{a\in\mathcal{A}}\sum_{h\in\mathcal{H}} w^s \cdot y^s_{ahj} \cdot \ln\left(p_{ah}+\varepsilon\right)$$

2

$$\text{s.t.} \quad \sum_{a\in\mathcal{A}} \eta_{ag} \sum_{h\in\mathcal{H}} y^s_{ahj} = x^s_{jg} \qquad \forall j\in\mathcal{J}, g\in\mathcal{G}, s\in\mathcal{S}$$

$$\sum_{a\in\mathcal{A}}\sum_{h\in\mathcal{H}} y^s_{ahj} = 1 \qquad \forall j\in\mathcal{J}, s\in\mathcal{S}$$

$$\sum_{j\in\mathcal{J}}\sum_{a\in\mathcal{A}}\sum_{h\in\mathcal{H}} y^s_{ahj}\cdot c_h = C \qquad \forall s\in\mathcal{S}$$

$$\sum_{j\in\mathcal{J}} x^s_{jg} = d^s_g \qquad \forall g\in\mathcal{G}, s\in\mathcal{S}$$

$$\sum_{g\in\mathcal{G}_k}\sum_{j\in\mathcal{J}\setminus\mathcal{J}_k} x^s_{jg} = 0 \qquad \forall k\in\tilde{\mathcal{K}}, s\in\mathcal{S}$$

$$\sum_{h\in\mathcal{H}} z_{hj} = 1 \qquad \forall j\in\mathcal{J}$$

$$y^s_{ahj} \le z_{hj} \qquad \forall a\in\mathcal{A}, h\in\mathcal{H}, j\in\mathcal{J}, s\in\mathcal{S}$$

1

$$z_{hj}\in\{0,1\} \qquad \forall h\in\mathcal{H}, j\in\mathcal{J}$$

$$x^s_{jg}\in\mathbb{N}_0 \qquad \forall j\in\mathcal{J}, g\in\mathcal{G}, s\in\mathcal{S}$$

$$y^s_{ahj}\in\{0,1\} \qquad \forall a\in\mathcal{A}, h\in\mathcal{H}, j\in\mathcal{J}, s\in\mathcal{S}$$

Koppka et al. (2018): Optimal distribution of operating hours over operating rooms using probabilities.

Prescriptive Analytics

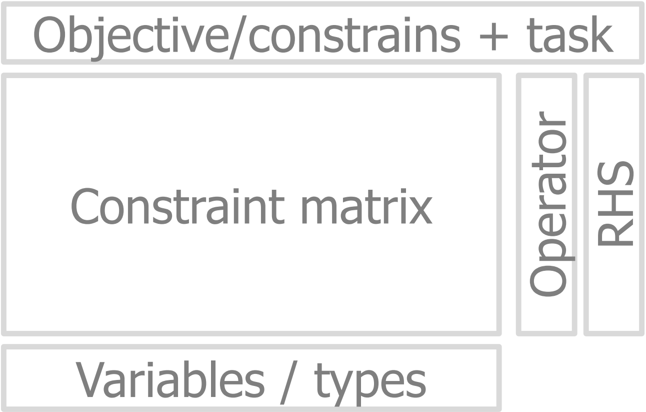
Optimisation in a nutshell

- Standard form of optimisation model:

$$c_1 \cdot x_1 + c_2 \cdot x_2 + \ldots + c_n \cdot x_n \rightarrow \max$$

$$\begin{aligned}
a_{11}x_1 + a_{12}x_2 + \ldots a_{1n}x_n &\leq b_1 \\
a_{21}x_1 + a_{22}x_2 + \ldots a_{2n}x_n &\leq b_2 \\
&\vdots \\
a_{m1}x_1 + a_{m2}x_2 + \ldots a_{mn}x_n &\leq b_m
\end{aligned}$$

$$x_1, x_2, \ldots, x_n \geq 0$$



Prescriptive Analytics

Optimisation in a nutshell

- Standard form of optimisation model:

$$c_1 \cdot x_1 + c_2 \cdot x_2 + \ldots + c_n \cdot x_n \to \max$$

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \ldots a_{1n}x_n &\leq b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots a_{2n}x_n &\leq b_2 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \ldots a_{mn}x_n &\leq b_m \end{aligned}$$

$$x_1, x_2, \ldots, x_n \geq 0$$

Typical structure:

$$c^t x$$

$$Ax \leq b$$

$$x \geq 0$$

Prescriptive Analytics

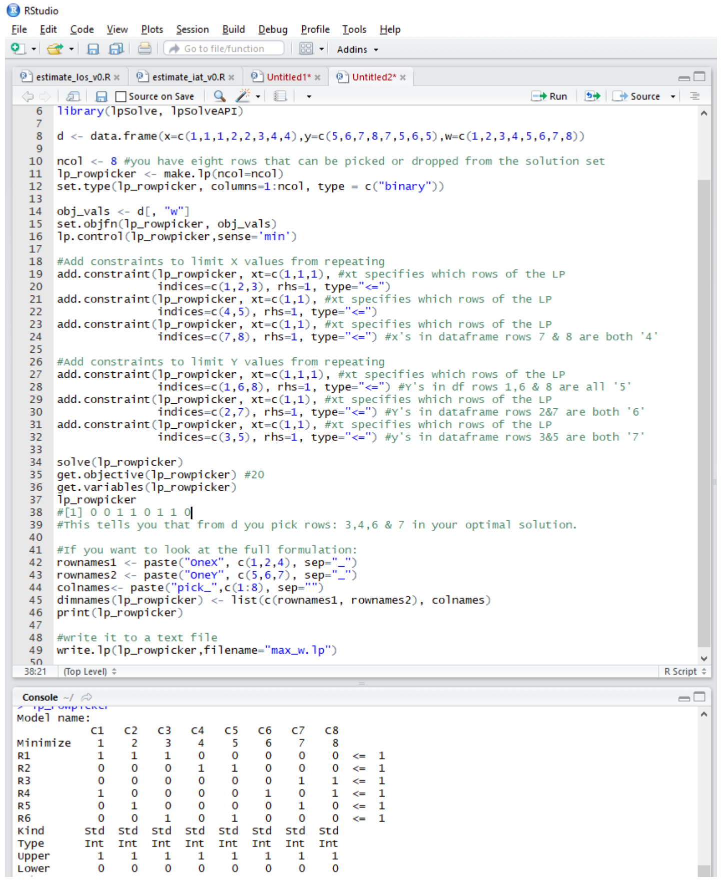
Optimisation in a nutshell

- Optimisation solvers use existing structure of problems/models
  - Commercial solvers: very fast, very expensive
  - Open source solvers: freely available, not so fast
  - MS Excel’s built-in solver: easy to access, limited in scope
- Apply solution techniques (i.e. algorithms) to provide optimal solution
- Interfaces
  - Programming languages
  - Data import/export (Excel, csv, etc.)

Prescriptive Analytics

Optimisation in a nutshell

- Transform models, if necessary
- Translate models: solver language
- Solve mathematical model
  - Use predefined algorithms, usually available through software package (such as simplex algorithm,
  - Develop new solution procedures
- Analyse solutions obtained for mathematical model
- Translate solutions to real-world problem



Prescriptive Analytics for Health Care

Prescriptive Analytics

Classification of decision problems

- *Distinction by time horizon:*
  - Strategic
  - Tactical
  - Operational (online / offline)
- *Distinction by managerial area:*
  - Medical
  - Resources/Material
  - Finance

Prescriptive Analytics

	Medical planning	Resource capacity planning	Materials planning	Financial planning
Strategic	Research, development of medical protocols	Case mix planning, capacity dimensioning, workforce planning	Supply chain and warehouse design	Investment plans, contracting with insurance companies
Tactical	Treatment selection, protocol selection	Block planning, staffing, admission planning	Supplier selection, tendering	Budget and cost allocation
Offline operational	Diagnosis and planning of an individual treatment	Appointment scheduling, workforce scheduling	Materials purchasing, determining order sizes	DRG billing, cash flow analysis
Online operational	Triage, diagnosing emergencies and complications	Monitoring, emergency coordination	Rush ordering, inventory replenishing	Billing complications and changes

← managerial areas →

↑ hierarchical decomposition ↓

Source: Hans/van Houdenhoven/Hulshof (2012), p. 311.

Prescriptive Analytics

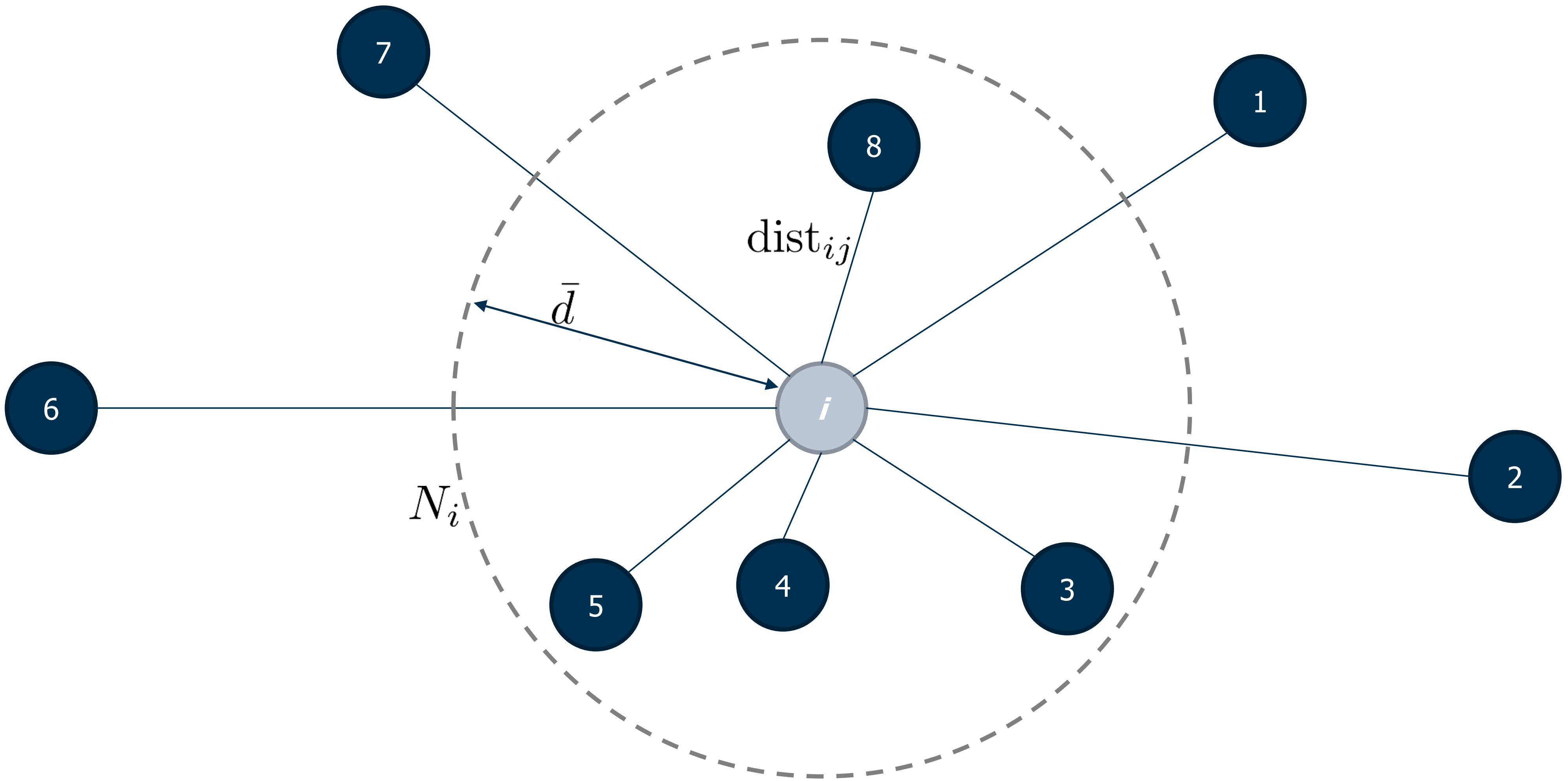
- Many possible applications:
  - Staffing (how many, rotas, emergency assignments, etc.)
  - Locations (where/how many, capacity allocation, re-allocation, etc.)
  - Operating room (size, departmental allocation, selection, sequence, etc.)
  - Blood Supply Chain
  - Hospital Layout

- Focus on two areas:
  1. Location analysis
  2. Operating room planning

Schumpeter School of Business and Economics

- Set covering location problem (SCLP, Toregas et al. 1971)
- Idea: What is the minimum number of locations so that every demand area can be "reached"?
- Key parts of the model:
  - Possible supply sites (=locations): $J = \{1, 2, \ldots, n\}$
  - Possible demand sites: $I = \{1, 2, \ldots, m\}$
  - Distances between demand area and potential location: dist_{ij}
  - Neighbourhood: subset of all potential locations which can be reached within given limit

Prescriptive Analytics
Location Planning



Schumpeter School of Business and Economics

- Set covering location problem (SCLP)
- Idea: What is the minimum number of locations so that every demand area can be "reached"?
- Decision variable:

$$y_j = \begin{cases} 1 & \text{if location } j \text{ is chosen,} \\ 0 & \text{else.} \end{cases}$$

- Conditions to "reach" demand area: defined by neighbourhood set N_i
- Neighbourhood = close enough:

$$N_i := \{j \in J \mid \text{dist}_{ij} \leq \bar{d}\}$$

Set covering location problem (SCLP)

- Idea: What is the minimum number of locations such that every demand area can be "reached"?
- Optimisation model:

$$\sum_{j \in J} y_j \rightarrow \min!$$

Minimise number of locations

$$\text{s.t.} \sum_{j \in N_i} y_j \geq 1 \qquad \forall i \in I$$

Cover every demand region at least once

$$y_j \in \{0,1\} \qquad \forall j \in J$$

Choose location j (1=yes, 0=no)

Prescriptive Analytics
Location Planning

- Maximal covering location problem (MCLP, Church and ReVelle 1974):
  - Choose locations such that covered population is maximised (demand: a_i)
  - Limited number of locations: *P*
- Decision variables:

$$y_i = \begin{cases} 1 & \text{if demand in region } i \text{ is covered,} \\ 0 & \text{else.} \end{cases}$$

$$x_j = \begin{cases} 1 & \text{if location } j \text{ is chosen,} \\ 0 & \text{else.} \end{cases}$$

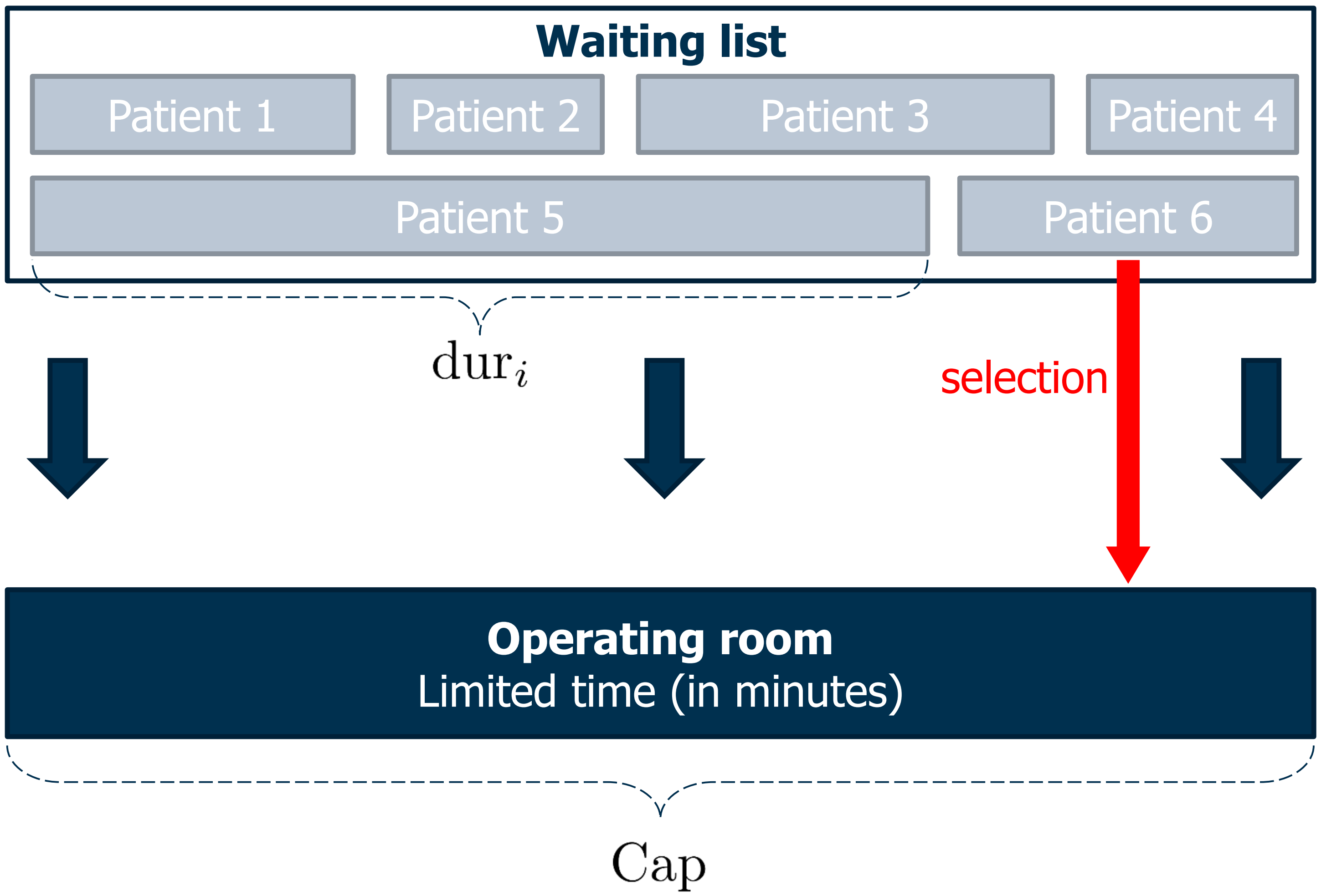
Prescriptive Analytics

Location Planning

- Maximal covering location problem (MCLP):
  - Choose locations such that covered population is maximised
  - Limited number of locations: *P*
- Optimisation model:

$\sum_{i=1}^{m} a_i \cdot y_i \rightarrow \max!$	Maximise amount of covered demand
s.t. $\sum_{j \in N_i} x_j \geq y_i \quad \forall i$	If location is chosen, demand is served
$\sum_{j=1}^{n} x_j = P$	Number of locations is limited
$x_j \in \{0,1\} \quad \forall j$ $y_i \in \{0,1\} \quad \forall i$	Binary decisions (area served, location chosen)

Prescriptive Analytics
Scheduling



Prescriptive Analytics
Scheduling

Schumpeter School of Business and Economics

- Select patients (index: i) from waiting list: $P = \{1, 2, \ldots, \bar{p}\}$
- Criteria:
  - As many patients as possible
  - Minimize idle time – target utilisation
  - Patients with longest waiting time first
- Decision variable:

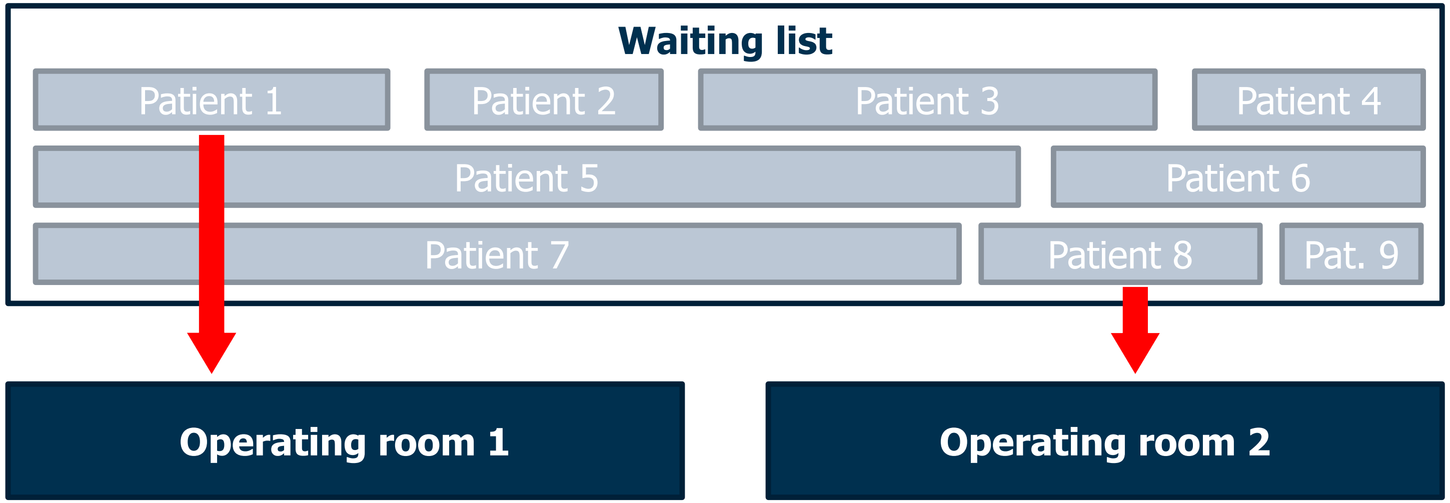
$$x_i = \begin{cases} 1 & \text{if patient is selected for surgery} \\ 0 & \text{else.} \end{cases}$$

- Surgery duration is known: dur_i
- Operating room has limited capacity: Cap

Prescriptive Analytics

Scheduling

- Extension 1: Plan multiple operating rooms simultaneously

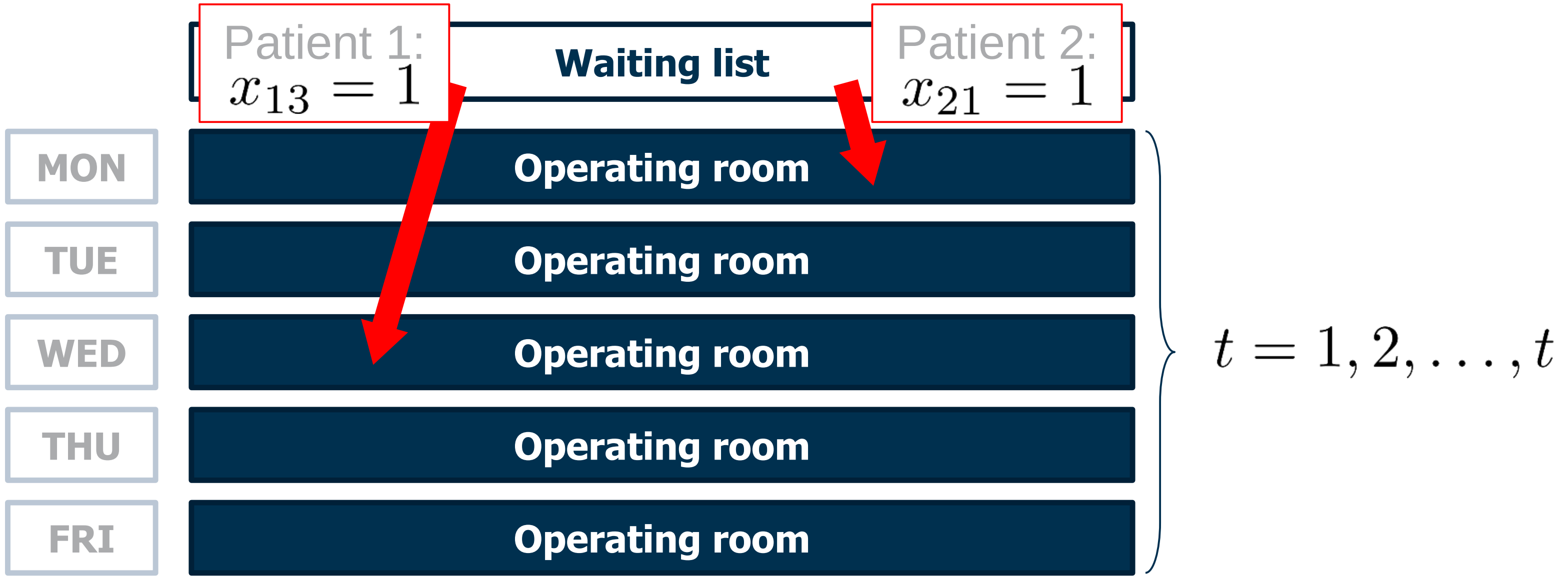


- Distinguish operating rooms: $J = \{1, 2, \dots, n\}$
- Potentially different capacities: Cap_j
- Decision variable:

$$x_{ij} = \begin{cases} 1 & \text{if patient } i \text{ is selected for room } j \\ 0 & \text{else.} \end{cases}$$

Prescriptive Analytics
Scheduling

- Extension 2: Plan a full week at once, five working days



- Decision variable:

$$x_{it} = \begin{cases} 1 & \text{if patient } i \text{ is selected for day } t \\ 0 & \text{else.} \end{cases}$$

Prescriptive Analytics

Scheduling

- Optimisation model: maximise number of patients in planning horizon

$\sum_{i=1}^{m} \sum_{t=1}^{T} x_{it} \rightarrow \max!$		Maximise #patients for the week
$\sum_{i=1}^{m} \mathrm{dur}_i \cdot x_{it} \leq \mathrm{Cap}$	$\forall t \in \{1, 2, \ldots, T\}$	Limited capacity, patients consume time in the OR
$\sum_{t=1}^{T} x_{it} \leq 1$	$\forall i \in \{1, 2, \ldots, m\}$	Schedule each patient at most once
$x_{it} \in \{0, 1\}$	$\forall i, t$	Scheduled: yes/no

Prescriptive Analytics

Which assumptions did we make?

- Ambulance locations:
  - Demand is known
  - Driving times are not changing
- Operating rooms:
  - Limited waiting list, equal importance of patients
  - Doctors are always available and can perform any surgery
  - Surgery duration is known, i.e. deterministic

What did we cover?

- Distinguish different types of Analytics
- Discuss potentials and limitations of optimisation
- Understand simple optimisation models
  - Ambulance location
  - Operating room planning

Literature

Textbooks

- Ozcan, Yasar A (2017): Analytics and Decision Support in Health Care Operations Management, 3rd Edition, Wiley.
- Brandeau, Margaret L.; Sainfort, Francois; Pierskalla, William P. (Eds.): Operations Research and Health Care: A Handbook of Methods and Applications, Springer.
- Hall, Randolph (2013): Patient Flow – Reducing Delay in Healthcare Delivery, Springer.
- Vissers, Jan; Beech, Roger (2005): Health Operations Management: Patient Flow Logistics in Health Care, Routledge.

Research articles

- Salmon, A.; Rachuba, S.; Briscoe, S.; Pitt, M. (2018): A structured literature review of simulation modelling applied to Emergency Departments, in: Operations Research for Health Care (in press)
- Rachuba, S.; Ashton, L.; Knapp, K.; Pitt, M. (2018): Streamlining pathways for minor injuries in Emergency Departments through radiographer-led discharge, in: OR for Health Care (in press)
- Rachuba, S.; Salmon, A.; Zhelev, Z.; Pitt, M. (2018): Redesigning the diagnostic pathway for chest pain patients in emergency departments, in: Health Care Management Science, 21(2):177-191.
- Rachuba, S.; Werners, B. (2017): A fuzzy multi-criteria approach for robust operating room schedules, in: Annals of Operations Research, 251(1):325-350.
- Rachuba, S.; Werners, B. (2014): A robust approach for scheduling in hospitals using multiple objectives, in: Journal of the Operational Research Society, 65:546-556.

Useful links

- Script: Optimisation with R:
  http://www.is.uni-freiburg.de/resources/computational-economics/5_OptimizationR.pdf
- Using lpSolveAPI with R:
  https://www.r-bloggers.com/linear-programming-in-r-an-lpsolveapi-example/
- Set Covering problem:
  http://mat.gsia.cmu.edu/classes/integer/node8.html
- Knapsack problem:
  http://mat.gsia.cmu.edu/classes/integer/node6.html#SECTION00032000000000000000
- Modeling and Solving LPs with R (free book):
  https://www.r-bloggers.com/modeling-and-solving-linear-programming-with-r-free-book/
- Introduction to glpkAPI with R:
  https://cran.r-project.org/web/packages/glpkAPI/vignettes/glpk-gmpl-intro.pdf
- Introduction to linear programming:
  https://www.math.ucla.edu/~tom/LP.pdf

CAUTION

All models are wrong but some are useful!

George E.P. Box (1919 - 2013)