Prescriptive Analytics in Health Care

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Economic Modelling in Health Care

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The first three of the following examples were discussed, modelled, and solved at the International Summer School in Riga this year. A fourth example has been added for which only the problem statement was given during the course. It is another opportunity to use mathematical optimisation to solve a health care specific problem. For all four examples, the optimisation model and the corresponding source code for `R` are provided.

Example A: Laboratory example

Consider two decision variables x_A (quantity of test A) and x_B (quantity of test B), with $x_A, x_B \geq 0$. The mathematical optimisation model to maximise the revenue while taking into account time requirements and availabilities can then be formulated as follows:

$$\max \quad 10 \cdot x_A + 10 \cdot x_B \tag{1}$$
$$\text{s.th.} \quad 2 \cdot x_A + x_B \leq 10 \tag{2}$$
$$x_A + 2 \cdot x_B \leq 8 \tag{3}$$
$$x_A, x_B \geq 0 \tag{4}$$

In the model above, the objective function (1) describes the total revenue if a certain combination of x_A and x_B is chosen. The task is to maximise this, i.e. find the highest possible value for the total revenue. Maximising the revenue, however, is constrained by (2) – (4). Restriction (2) considers the limited time (10 hours) for which the senior assistant is available. In (3) the time limitations of the junior assistant are taken into account respectively. Finally, (4) ensures that only non-negative quantities can be chosen.

Source code for R: Please see the file `example_a.R` for a coded version of this using the library `lpSolve`.

Example B: Location planning using the SLCP model

Consider in this example 11 residential areas ($i = 1, 2, \ldots, 11$) and in this case also 11 possible locations for the ambulances ($j = 1, 2, \ldots, 11$). The decision variable is therefore defined as:

$$x_j = \begin{cases} 1, & \text{if ambulance is placed in location } j \\ 0, & \text{else.} \end{cases} \tag{5}$$

For every residential area i, the possible locations[1] for ambulances can be listed as follows:

N_1:	1,2,3,4	N_7:	4,6,7,8
N_2:	1,2,3,5	N_8:	5,6,7,8,9,10
N_3:	1,2,3,4,5,6	N_9:	5,8,9,10,11
N_4:	1,3,4,6,7	N_{10}:	8,9,10,11
N_5:	2,3,5,6,8,9	N_{11}:	9,10,11
N_6:	3,4,5,6,7,8		

Using this information, the optimisation model to minimise the number of ambulances so that every residential area can be reached by an ambulance can be formulated as follows:

$$\begin{array}{lccccccccccccc}
\min & x_1 & +x_2 & +x_3 & +x_4 & +x_5 & +x_6 & +x_7 & +x_8 & +x_9 & +x_{10} & +x_{11} & & \\
\text{s.th.} & x_1 & +x_2 & +x_3 & +x_4 & & & & & & & & \geq & 1 \\
& x_1 & +x_2 & +x_3 & & +x_5 & & & & & & & \geq & 1 \\
& x_1 & +x_2 & +x_3 & +x_4 & +x_5 & +x_6 & & & & & & \geq & 1 \\
& x_1 & +x_2 & +x_3 & +x_4 & & +x_6 & +x_7 & & & & & \geq & 1 \\
& & x_2 & +x_3 & & +x_5 & +x_6 & & +x_8 & +x_9 & & & \geq & 1 \\
& & & x_3 & +x_4 & +x_5 & +x_6 & +x_7 & +x_8 & & & & \geq & 1 \\
& & & & x_4 & & +x_6 & +x_7 & +x_8 & & & & \geq & 1 \\
& & & & & x_5 & +x_6 & +x_7 & +x_8 & +x_9 & +x_{10} & & \geq & 1 \\
& & & & & x_5 & & & +x_8 & +x_9 & +x_{10} & +x_{11} & \geq & 1 \\
& & & & & & & & x_8 & +x_9 & +x_{10} & +x_{11} & \geq & 1 \\
& & & & & & & & & x_9 & +x_{10} & +x_{11} & \geq & 1
\end{array}$$

$x_j \in \{0, 1\}$ for all $j = 1, 2, \ldots, 11$

Source code for R: Please see the file `example_b.R` for a coded version of this using the library `lpSolve`.

[1] Note: A residential area can be covered if the ambulance is placed in the residential area itself or in an adjacent area.

Example C: Operating room planning
The task is to select one out of 15 patients ($i = 1, 2, \ldots, 15$) to be scheduled for surgery. The binary decision variable is defined as:

$$x_i = \begin{cases} 1, & \text{if patient } i \text{ is selected for surgery} \\ 0, & \text{else.} \end{cases} \tag{6}$$

The task is to maximise the number of surgeries to be selected for surgery such that the available time in the operating room is not exceeded. The optimisation model can be formulated as follows:

$$\begin{aligned} \max \quad & x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13} + x_{14} + x_{15} && (7) \\ \text{s.th.}^2\text{:} \quad & 35x_1 + 17x_2 + 48x_3 + 23x_4 + 121x_5 + 230x_6 + 47x_7 + 87x_8 + 134x_9 + \ldots \\ & \ldots + 55x_{10} + 29x_{11} + 90x_{12} + 65x_{13} + 40x_{14} + 76x_{15} \leq 480 && (8) \\ & x_i \in \{0, 1\} \text{ for all } i = 1, 2, \ldots, 15 && (9) \end{aligned}$$

Source code for R: Please see the file `example_c.R` for a coded version of this using the library `lpSolve`.

Example D: Shift planning[3]

The basic idea with shift planning is to minimise the number of staff members starting to work on a particular day (Mon=1, Tue=2, ..., Sun=7). When a staff member starts to work, e.g. on Monday, this member of staff will work on 5 consecutive days (i.e. Mon, Tue, Wed, Thu, Fri) followed by two days off (Sat, Sun). If the first day would Tuesday, this member of staff would Work Tuesday to Saturday, but Sunday and Monday would be days off. In fact, this 5−day pattern can be started on any day of the week. We therefore have seven different patterns and are interested in the number of people starting to work on a particular day. We use the integer decision variable $x_t \in \{0, 1, 2, 3 \ldots\}$ if an employee starts to work on day $t = 1, 2, \ldots, 7$ (with 1=Mon, 2=Tue, etc.) and works for 5 days followed by two days off. If somebody is working, 8 hours of working time are provided and respectively 0 hours for a day off. For every day, the working hours provided need to be greater or equal to the required minimum time. The optimisation model can be formulated as follows:

$$\begin{array}{lllllllllll} \min & x_1 & +x_2 & +x_3 & +x_4 & +x_5 & +x_6 & +x_7 & +x_8 & & \\ \text{s.th.} & 8x_1 & & & +8x_4 & +8x_5 & +8x_6 & +8x_7 & +8x_8 & \geq & 48 \\ & 8x_1 & +8x_2 & & & +8x_5 & +8x_6 & +8x_7 & +8x_8 & \geq & 48 \\ & 8x_1 & +8x_2 & +8x_3 & & & +8x_6 & +8x_7 & +8x_8 & \geq & 52 \\ & 8x_1 & +8x_2 & +8x_3 & +8x_4 & & & +8x_7 & +8x_8 & \geq & 52 \\ & 8x_1 & +8x_2 & +8x_3 & +8x_4 & +8x_5 & & & +8x_8 & \geq & 36 \\ & 8x_1 & +8x_2 & +8x_3 & +8x_4 & +8x_5 & +8x_6 & & & \geq & 24 \\ & & 8x_2 & +8x_3 & +8x_4 & +8x_5 & +8x_6 & +8x_7 & & \geq & 24 \\ & & & 8x_3 & +8x_4 & +8x_5 & +8x_6 & +8x_7 & +8x_8 & \geq & 99 \\ & \multicolumn{10}{l}{x_t \in \{0, 1, 2, \ldots\} \text{ for all } t = 1, 2, \ldots, 7} \end{array}$$

Optimal objective function value: 8.
Variable solutions: $x_1 = 4, x_2 = 1, x_3 = 1, x_5 = 1, x_7 = 1$ and $x_4 = x_6 = 0$.

Solution: The minimum number of staff is 8 people. 4 people start on Monday and 1 person on each of the following days: Tuesday, Wednesday, Friday, and Sunday.

Source code for R: Please see the file `example_d.R` for a coded version of this using the library `lpSolve`.

[3]This was not discussed in the tuorial, but for those who are interested in solving this, an idea how to solve this, an optimisation model and a code file to obtaion the solution are provided.